**Tutorial Activity 5**

**Week 6**

1. Explain what stylised shapes would be expected for the autocorrelation and partial autocorrelation functions for the following stochastic processes:
2. white noise
3. an AR(2)
4. an MA(1)
5. an ARMA (2,1).

|  |  |  |
| --- | --- | --- |
| Process | acf | pacf |
| White noise | No significant coefficients | No significant coefficients |
| AR(2) | Geometrically declining or damped sinusoid acf | First 2 pacf coefficients significant, all others insignificant |
| MA(1) | First acf coefficient significant, all others insignificant | Geometrically declining or damped sinusoid pacf |
| ARMA(2,1) | Geometrically declining or damped sinusoid acf | Geometrically declining or damped sinusoid pacf |

A couple of further points are worth noting. First, it is not possible to tell what the signs of the coefficients for the acf or pacf would be for the last three processes, since that would depend on the signs of the coefficients of the processes. Second, for mixed processes, the AR part dominates from the point of view of acf calculation, while the MA part dominates for pacf calculation.

1. Consider the following ARMA process.
2. Determine whether the MA part of the process is invertible.

The important point here is to focus on the MA part of the model and to ignore the AR dynamics. The characteristic equation would be

(1+0.58*z*) = 0

The root of this equation is -1/0.58 = -1.72, which lies outside the unit circle, and therefore the MA part of the model is invertible.

1. What procedure might be used to estimate the parameters of an ARMA model? Explain, briefly, how such a procedure operates, and why OLS is not appropriate.

Moving average and ARMA models cannot be estimated using OLS – they are usually estimated by maximum likelihood. Autoregressive models can be estimated using OLS or maximum likelihood. Pure autoregressive models contain only lagged values of observed quantities on the RHS, and therefore, the lags of the dependent variable can be used just like any other regressors. However, in the context of MA and mixed models, the lagged values of the error term that occur on the RHS are not known *a priori*. Hence, these quantities are replaced by the residuals, which are not available until after the model has been estimated. But equally, these residuals are required in order to be able to estimate the model parameters. Maximum likelihood essentially works around this by calculating the values of the coefficients and the residuals at the same time. Maximum likelihood involves selecting the most likely values of the parameters given the actual data sample, and given an assumed statistical distribution for the errors. This technique will be discussed in greater detail in the section on volatility modelling in Chapter 9.